# CLEAR IDENTIFICATION OF FUNDAMENTAL IDEA OF NAKAMURA'S TECHNIQUE AND ITS APPLICATIONS

**Yutaka NAKAMURA**[1]


## SUMMARY

A method which employs microtremor has been introduced for estimating dynamic characteristics of surface layers, in early 1950. Then usage of this method has received lots of criticism considering uncertainty about source of microtremor. After an introduction of the Nakamura's technique (H/V or QTS technique; Nakamura, 1989), many people have paid a renewed great attention for estimating dynamic characteristics of ground and structures using microtremor, since clear and reliable information was provided by very simple and inexpensive noise measurements.

In recent years, although several researchers claimed that theoretical ground of this technique is not clear and consensus based on experiment couldn't be reached, there have been many successful experimental studies based on these technique. Many theoretical studies have been performed, for explaining the amount of types of waves included in microtremor and checking the applicability of the QTS technique. And some of them are suggested that the peak on H/V ratio can be explained with the fundamental peak of Rayleigh waves. From the output of these researches, explanation of microtremor with Rayleigh waves caused some confusion between users and the author decided to clear out this problem.

The basic idea and the main goal of QTS technique are tried to be re-explained in present paper. The author's explanation about the effects of contents of Rayleigh waves in microtremor is also given. Other possible usage of products from QTS technique (predominant frequency and amplification factor) for hazard estimation is also given. As it is well known, occurrence of earthquake damage depends upon strength, period and duration of seismic motions. And these parameters are strongly influenced by seismic response characteristics of surface ground and structures. This reality makes investigation of vulnerability of ground and structures an important issue, before the earthquake occurs. For this purpose, vulnerability indices called K values were proposed by Nakamura (1996). K values are simply derived from strains of ground and structures. Formulation of K values for ground (Kg) and some application examples are also given in present paper. These new values give a chance to estimate vulnerabilities of all types of structures and ground, before the real damage occurs.


## INTRODUCTION

Damages caused by the recent earthquakes are concluded as a direct result of local geological conditions affecting the ground motion. Best approach for understanding ground conditions is through direct observation of seismic ground motion, but such studies are restricted to areas with relatively high rates of seismicity. Because of these restrictions in other methods, such as high rates of seismicity and the availability of an adequate reference site, non-reference site methods have been applied to site response studies. Microtremor is a very convenient tool to estimate the effect of surface geology on seismic motion without needing other geological information.

H/V (or QTS, Quasi-Transfer Spectra) technique fits very well to this description and it has received great attention from all over the world with its simplicity together with quick information about dynamic characteristics of ground and structures. Although several researchers claimed that theoretical background of this technique is not clear, there have been many successful experimental studies performed. Method is attractive since it gives the ease of data collection and it can be applied in areas of low or even no seismicity.

[1] *System and Data Research Co. Ltd., 3-25-3 Fujimidai, Kunitachi-shi, Tokyo, Japan yutaka@sdr.co.jp*

H/V technique was developed by author with relating bore hole investigations together with strong motion records analysis, on the various geological site conditions. It was hypothesized that the vertical component of ambient noise keeps the characteristics of source to sediments surface ground, is relatively influenced by Rayleigh wave on the sediments and can therefore be used to remove both of the source and the Rayleigh wave effects from the horizontal components. It is effective to identify the fundamental resonant frequency of a sedimentary layer, with implied amplification factors that are more realistic than those obtained from sediment to rock site ratios. It has been shown by many researchers (ex. Ohmachi et. al., 1991; Lermo et. al., 1992; Field and Jacob, 1993, 1995) that how such H/V ratio of noise can be used to identify the fundamental resonant frequency and amplification factor of sediments.

Looking to the examples in the study of Nogoshi and Igarashi (1971) which compared H/V of Rayleigh wave with that of microtremors and concluded that microtremor was mostly composed of Rayleigh wave, some of theoretical studies (Lachet and Bard, 1994; Konno and Ohmachi, 1998; Bard, 1998) suggested that the peak on H/V can be explained with the fundamental mode of Rayleigh waves. If we think that this approach is true, microtremor should be considered to consist of only Rayleigh wave. On the other hand, if we check the examples given on Nogoshi and Igarashi (1971) carefully (which will be discussed later), we can clearly see that, at the peak frequency of H/V of Rayleigh wave, the energy of Rayleigh wave is very small, nearly close to the zero. Rayleigh wave has its maximum energy at near trough frequency of H/V. Because of this, peak of H/V cannot be explained by Rayleigh wave energy. As it is explained by Nakamura(1989), H/V of microtremor at peak frequency range can be explained with vertical incident SH wave.

## 2. WAVES IN MICROTREMOR AND CHARACTERISTICS OF STRONG GROUND MOTION

Many observations and experiences on microtremor records show that microtremor consists of both body and surface waves, but there is no established theory concerning what kinds of wave motions the microtremor is made up from. In Nakamura (1989), the purpose of the author was the estimation of the amplification factor caused by multiply refracted vertical incident SH waves. For this purpose, Rayleigh wave contained by microtremor was considered as noise and eliminated inside the H/V process.

Seismic experiences shows that strong motion at the rock site in horizontal and vertical components does not show much differences. Also if we take the H/V ratio on the record measured simultaneously at the ground surface and basement, we can see that maximum acceleration ratio is consistent as shown in Figure 1.

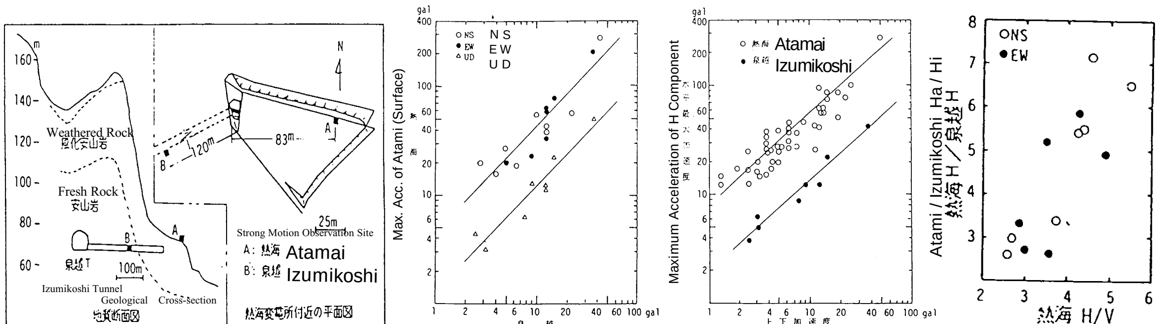



**Figure 1. Relation of Maximum accelerations on surface and basement (Nakamura and Saito, 1983)**

Figure 2 shows the difference in records caused by difference in earthquakes and observation sites. As we can follow, records in different stations even for the same earthquake are different because of the different site characteristics. On the other hand, for different earthquakes same site gives almost same type of records. In other words, it may be said that, affecting dynamic characteristics, the effect of surface layer is most critical among the other factors.

For the investigation of dynamic characteristics, boring is one of the most accurate method, but it is costly and time consuming and is not available all the time. If we measure both at surface Hf and basement Hb and take the ratio we can get the transfer function. Method which employs microtremor is well known and there may be some cases that transfer function calculated from Hf/Hb ratio and from velocity model of boring data show some differences. The reason for this is basically the evidence of surface waves. In practice Rayleigh wave should be eliminated to estimate transfer function correctly. H/V method (Nakamura, 1989) was introduced for this purpose and verified with both strong ground motion and microtremor data.

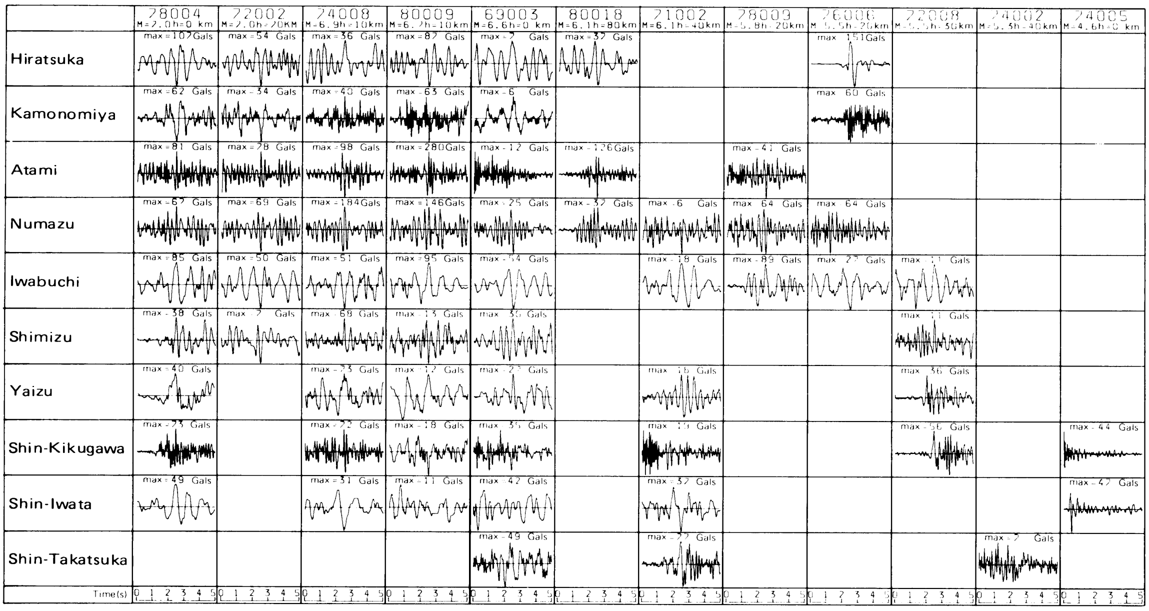



**Figure 2. Difference in Accelerogram due to difference in earthquake and observation stations (Nakamura, 1989)**

## 3. RELATION BETWEEN H/V SPECTRUM OF MICROTREMOR AND H/V OF RAYLEIGH WAVES

Nakamura (1989), gives theoretical definition of the H/V technique with multiple refraction of SH waves. On the other hand, there is a group of researchers who try to explain the peak of H/V ratio with the evidence of Rayleigh waves. As it is mentioned before, microtremor consists of many kinds of waves. Link to the surface waves is based on the assumption that noise predominantly consists of surface waves. Under this assumption Bard (1998) explains that, many researchers are agree on following two arguments; First, H/V is basically related to the ellipticity of Rayleigh waves because of the predominance of Rayleigh waves in vertical component. Second, this ellipticity is frequency dependent and exhibits a sharp peak around the fundamental frequency for sites displaying a high enough impedance contrast between the surface and deep materials. This approach, simply comes from the similarity of the figures of H/V ratio of microtremor and H/V of fundamental mode of Rayleigh waves, but just looking to this similarity this conclusion cannot be reached.

Nogoshi and Igarashi (1971) gives examples on different sites. Figure 3 shows two of these examples for the microtremor records in Hakodate, Japan. It can be clearly seen from these figures that, energy of Rayleigh wave does not appear on the peak of H/V of Rayleigh wave. We can easily see that there is no energy around the peak frequency of H/V and amplitude is almost zero for horizontal and zero for vertical

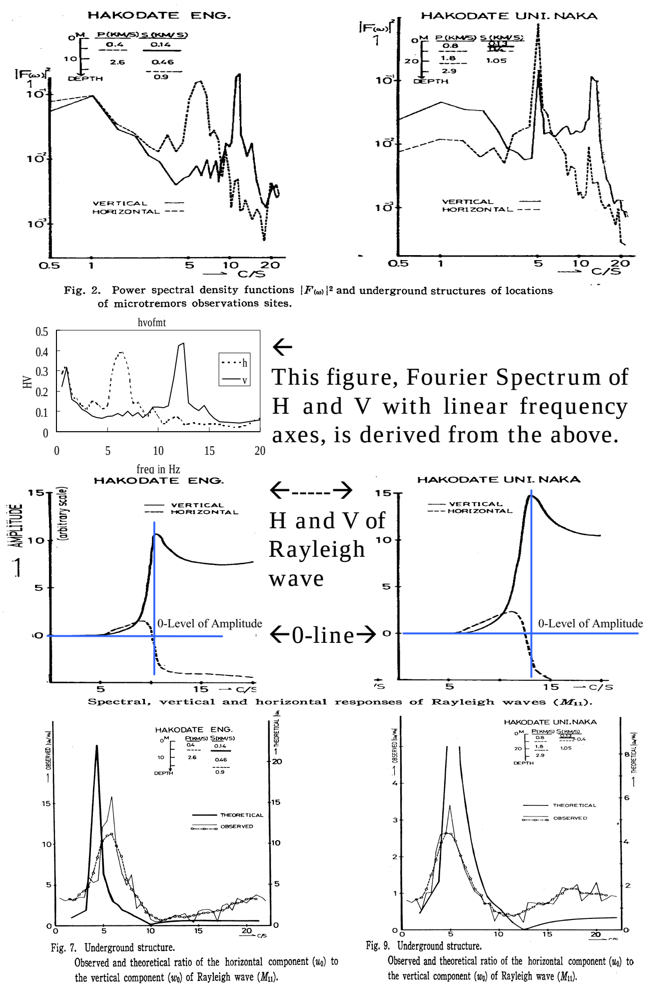



**Figure 3. Relation of H, V and H/V for microtremor and for Rayleigh wave (Nogoshi & Igarashi, 1971)**

components of Rayleigh waves. On the other hand, Rayleigh wave energy gets its maximum on later frequencies at minimum group velocity of Rayleigh wave and this is nearly equal to trough frequency which is almost two times of the H/V peak frequency.

For different wave-lengths, Ohta (1962) calculated H/V and phase velocity of Rayleigh waves for two layers model for various impedance ratio (varying between 1.2-4.5) and Poisson's ratio (varying between 0.25-0.49) both in sedimentary and basement layers. By using these calculation results, H/V of Rayleigh wave versus frequency is drawn in Figure 4 to show the relations between impedance, peak and frequency. And frequency in this figure is normalized with Cs/4h, Cs is S wave velocity and h is depth of sediment. Group velocities calculated from phase velocities are normalized with S wave velocity and drawn with the frequency normalized with Cs/4h. Figure 5 shows the distribution of group velocity versus frequency. As we can follow from this figure, if impedance ratio is less than two, energy of Rayleigh wave distributes to the lower frequencies. Figure 6 shows the change of impedance ratio for frequencies of trough and peak of H/V and minimum group velocity of Rayleigh waves. For trough and minimum group velocity (for almost all impedance ratio values) frequency changes between 1.5-2. On the other hand, only for the peak frequency varies in a wider range for different impedance ratio. The energy of Rayleigh wave is almost zero at peak frequency of H/V, and at the trough frequency of H/V the energy becomes to maximum. When the impedance ratio is greater than 2.5, Rayleigh wave does not affect to the H/V peak of ground motion . And on the other hand, when the impedance ratio is less than 2.5, Rayleigh wave affects to the H/V peak of ground motion. It is consistent with the experience of many researchers measuring microtremor. Figure 7 shows the change of H/V ratio of peak and trough of Rayleigh wave versus impedance ratio. In this figure, for impedance ratio around two, there is not a big difference in the H/V of Rayleigh wave. For the impedance ratio bigger than two, H/V ratio for peak of Rayleigh wave increases immediately. It seems impossible to estimate the impedance ratio from the H/V peak of Rayleigh wave. All these examples prove that, as it was explained by Nakamura(1989), H/V peak is mostly related with SH wave and cannot be explained with the Rayleigh waves.

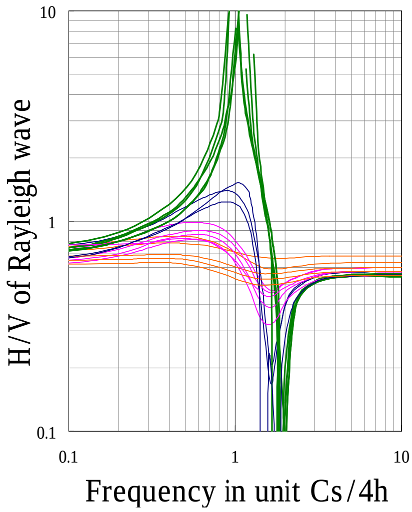



**Figure 4. H/V of Rayleigh wave for two layered ground**

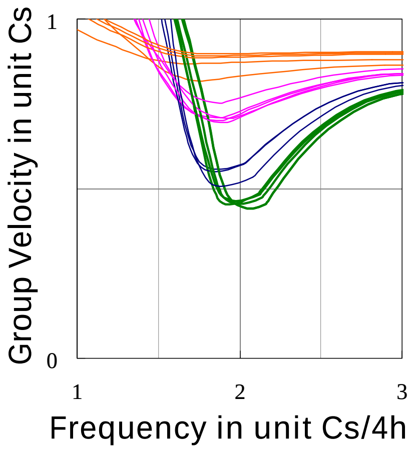



**Figure 5. Group velocity of Rayleigh waves**

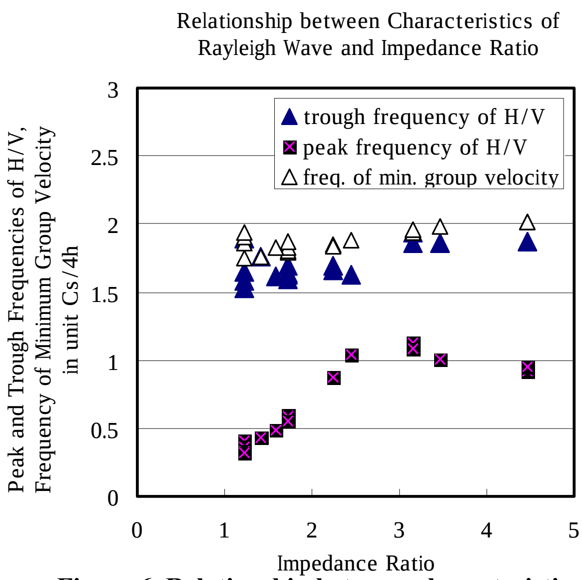



**Figure 6. Relationship between characteristics of Rayleigh wave and impedance ratio.**

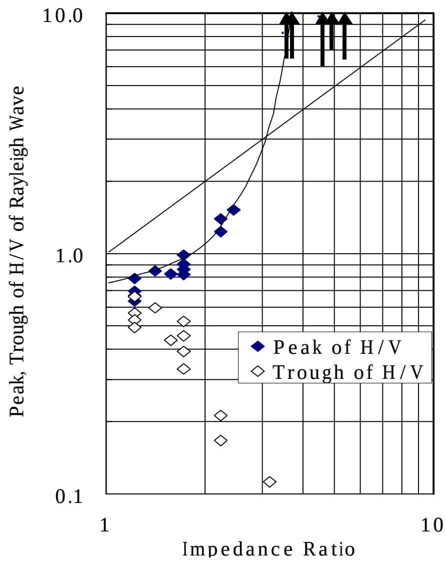



**Figure 7. Relationship between peak and trough of H/V of Rayleigh wave and impedance ratio**

## 5. EXPLANATION ON H/V RATIO (QTS) TECHNIQUE

Figure 8, shows the typical geological structure of sedimentary basin. Definition of ground motions and their spectra at different places are defined in following lines. Here microtremor is divided into two parts considering it contains Rayleigh wave and other waves. Then, horizontal and vertical spectra on the surface ground of the sedimentary basin (Hf, Vf) can be written as follows.

$$H_f = A_h * H_b + H_s \qquad V_f = A_v * V_b + V_s \qquad (1)$$

$$T_h = \frac{H_f}{H_b} \qquad T_v = \frac{V_f}{V_b} \qquad (2)$$

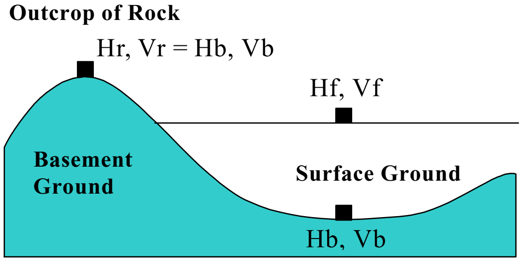



**Figure 8. Typical geological structure of sedimentary basin**

where Ah and Av are amplification factor of horizontal and vertical motions of vertically incident body wave. Hb and Vb are spectra of horizontal and vertical motion in the basement under the basin (outcropped basin). Hs and Vs are spectra of horizontal and vertical directions of Rayleigh waves. Th and Tv are amplification factor of horizontal and vertical motion of surface sedimentary ground based on seismic motion on the exposed rock ground near the basin. In general, P wave velocity is more than three-four times of S wave velocity. In such sedimentary layer, vertical component cannot be amplified (Av=1) around the frequency range where horizontal component receives large amplification. If there is no effect of Rayleigh waves, Vf ≅ Vb. On the other hand, if Vf is larger than Vb, it is considered as the effect of surface waves. Then estimating the effect of Rayleigh waves by Vf/Vb (=Tv), horizontal amplification can be written as,

$$T_h^* = \frac{T_h}{T_v} = \frac{\frac{H_f}{V_f}}{\frac{H_b}{V_b}} = \frac{QTS}{\frac{H_b}{V_b}} = \frac{\left[A_h + \frac{H_s}{H_b}\right]}{\left[A_v + \frac{V_s}{V_b}\right]} \qquad (3)$$

where ,

$$QTS = \frac{H_f}{V_f} = \frac{A_h * H_b + H_s}{A_v * V_b + V_s} = \frac{H_b}{V_b} \cdot \frac{\left[A_h + \frac{H_s}{H_b}\right]}{\left[A_v + \frac{V_s}{V_b}\right]} \qquad (4)$$

In equation (4), $H_b/V_b \cong 1$. Hs/Hb and Vs/Vb are related with the route of energy of Rayleigh waves. If there is no influence of Rayleigh wave, QTS = Ah/Av. If amount of Rayleigh wave is high, then second term in above formulation gets dominant and QTS = Hs/Vs and the lowest peak frequency of Hs/Vs is nearly equal to the lowest proper frequency Fo of Ah (see, Figure 6). In the range of Fo, Av=1. QTS shows stable peak at frequency Fo. Even when influence of Rayleigh wave is large, Vs become small (which results in a peak of Hs/Vs) around the first order proper frequency due to the multiple reflection of horizontal motions. And QTS = Ah, if microtremors of the basement Vb is relatively large comparing to the Rayleigh wave. Briefly, QTS represents the first order proper frequency due to multiple reflection of SH wave in the surface ground layer and resulted amplification factor, regardless of the influence degree of Rayleigh waves.

Figure 9, shows a schematic comparison of Horizontal ($H_f$), Vertical ($V_f$), $H_f/H_b$ (spectral ratio of sediment site to the reference site) and $H_f/V_f$ (H/V technique). As it can be followed QTS is smaller than the theoretical transfer function. Since $H_f$ includes the effects of Rayleigh waves, $H_f/H_b$ is bigger than the theoretical transfer function.

If influence of Rayleigh wave becomes larger, QTS<1 in the wide range of frequency and if influence is small, QTS locally expected to be smaller than one, in the narrow frequency range at frequency several times higher than Fo, because of the influence of vertical motion.

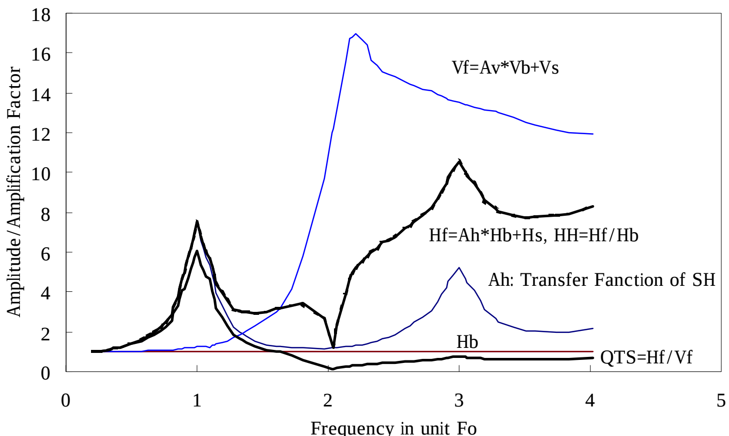



**Figure 9. Schematic comparison of Horizontal ($H_f$), Vertical ($V_f$), $H_f/H_b$ (spectral ratio of sediment site to the reference site)**

Main waves consisting of microtremors are either body waves or Rayleigh waves, or depending on the location and other conditions can be mixture of both waves. For such microtremors, if we calculate QTS, first order proper frequency by multiple reflections of SH wave in the surface layer and its amplification factor can be calculated correctly. More explanation regarding Rayleigh waves can be found in Nakamura(1996).

## 6. BASEMENT DEPTH RELATED WITH QTS

From above discussions we can now conclude that peak of QTS is caused by multiple refraction of S waves and QTS(Quasi Transfer Spectrum) represents the meaning coming from its name. On the other hand, here it should also be addressed the depth of the basement related with QTS. The frequency Fo related with QTS is;

$$F_0 = \frac{C_s}{4h} \qquad (5)$$

and the amplification factor A for this frequency is related with impedance ratio. If densities for basement and surface layer are same then,

$$A_0 = \frac{C_b}{C_s} \qquad (6)$$

and depth of basement h is,

$$h = \frac{C_b}{4A_0 \cdot F_0} \qquad (7)$$

$C_b$ is S wave velocity of basement.

Figure 10, shows the depth of basement along a Shinkansen line estimated from microtremor for the case of basement velocity $Cb$=600m/sec. The comparisons of these calculated values are compared with boring data and results showed that, alluvium-diluvium contact line is the basement of QTS.

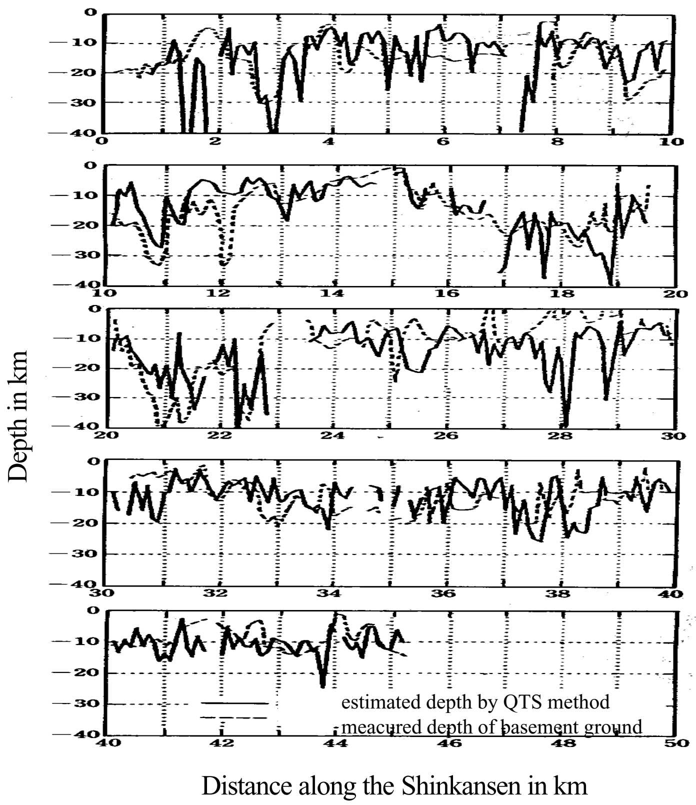
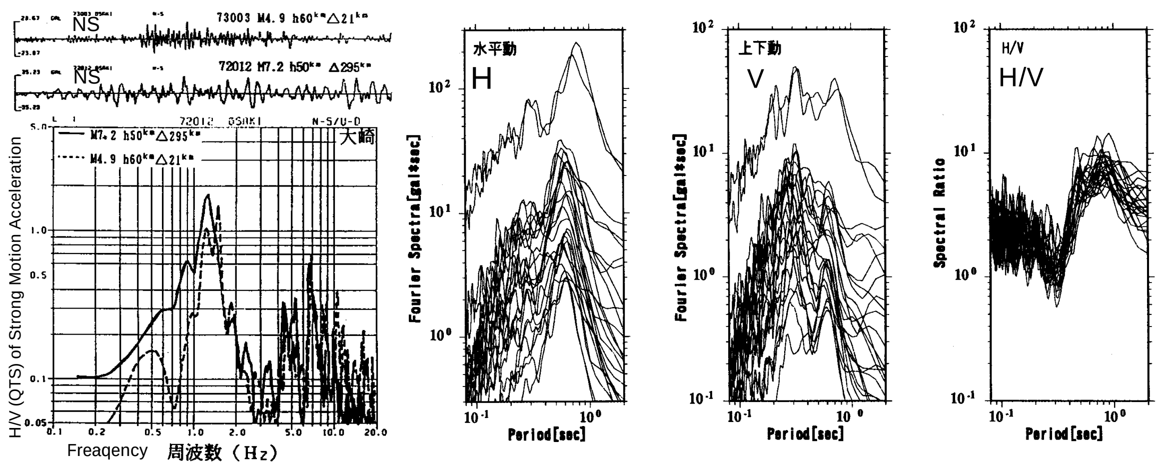



**Figure 10. Estimated depth from microtremor measurement along a Shinkansen (Nakamura et. al., 1990)**

## 7. H/V OF EARTHQUAKE DATA

Figure 11(a), shows H/V of strong ground motion of two earthquake different earthquake recorded at the same station. Although waveforms look different, when we take H/V, they look similar. Another example can be found also in a recent study of Okuma et. al. (1999) which is given in Figure 11(b). Results again shows that




(a) Osaki (Nakamura et. al., 1989) (b) Miyazaki (Okuma et. al. , 1999)

**Figure 11. H/V of strong ground motion for different earthquakes recorded at the same station.**

H/V gives similar characteristics for the different earthquakes recorded at the same stations. If we think that H/V of earthquake record can be explained by Rayleigh wave, we should see same characteristics for all directions of the station, but this is out of practice and it is proper to think that QTS of earthquake motion indicate the transfer function of local surface ground, as same as QTS of microtremor.

## 8. VULNERABILITY INDEXES, K-VALUES FOR SURFACE GROUND

K values have been proposed by Nakamura(1996) for accurately estimating earthquake damage of surface ground and structures. Here in this paper, only the formulation of Kg, K value for ground, will be given as an example of application of outputs from QTS method. For calculating Kg, shear strain of the ground is considered. Figure 12 simply shows the shear deformation of surface ground.

$$\gamma = A_g \cdot \frac{d}{h} \qquad (8)$$

where Ag is amplification factor of surface layer, h is thickness of surface layer and d is seismic displacement of the basement. Putting the S wave velocities of the basement and surface layer ($C_b$ and $C_f$) natural frequency F of the surface layer can be expressed as,

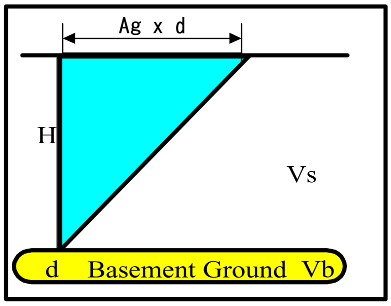



**Figure 12 . Surface ground deformation**

$$F_g = \frac{V_b}{4A_g \cdot h} \qquad (9)$$

Acceleration α in the basement can be written as

$$\gamma = \frac{A_g \cdot \alpha_b}{(2\pi F_g)^2} \cdot 4A_g \cdot \frac{F_g}{C_b} \qquad (10)$$

$$= \frac{A_g^2}{F_g} \cdot \frac{\alpha_b}{\pi^2 C_b}$$

$$= c \cdot K_g \alpha$$

$\alpha_b = (2\pi F_g)^2 d$ and shear strain γ is expressed as follows, where,

$$c = \frac{1}{\pi^2 \cdot v_b} \quad ; \quad K_g = \frac{A_g^{\ 2}}{F_g}$$

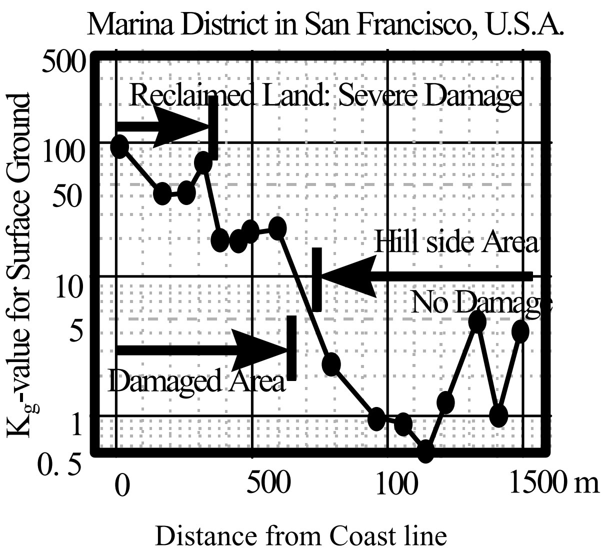



**Figure 13. Kg values calculated for Loma Prieta earthquake (Nakamura et. al., 1990)**

c is expected to be almost constant for various sites. Effective shear strain defined by e % of equation (8) becomes to nearly equal to the product of Kg and $\alpha_b$, under the assumptions of e = 60% and Cb = 600m/s. Kg is a value corresponding to the site and can be considered as a vulnerability index of the site, which might be useful to select weak points of ground.

K values for various types of structures and some application examples can be found in Nakamura (1996), Nakamura (1997), Nakamura and Gurler (1999).

## 9. CONCLUDING REMARKS

It is proved in the present paper that peak of H/V ratio (QTS), either for microtremor of for earthquakes cannot be explained with Rayleigh waves, since Rayleigh wave energy is very small for the peak frequency but high on the trough of H/V ratio. Author's explanation in Nakamura (1989) is correct for explaining this peak with SH waves. Vulnerability index, K value for ground which gives a chance to estimate the earthquake damage before the earthquake occurs is also briefly explained

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